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Fuzzy Programming Technique for Multiobjective Capacitated Transportaion Problem

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ABSTRACT
In much practical application, it is realistic to assume that the amount which can be sent on any particular route is restricted by the capacity of that route. Further, when a route is altogether excluded, this can be expressed by limiting its capacity zero; this is an alternative to attach a very high cost to that route. Here we discussed the variants of single objective solid transportation problem with capacity restrictions. We have developed a Fortran program based on fuzzy programming technique to solve the multiobjective solid capacitated transportation problems.
Key words: Solid capacitated transportation problem, Multiobjective Problem, Fuzzy programming technique.

I. INTRODUCTION

A transportation problem with capacity restrictions is a linear programming problem and can be solved by a LP algorithm. Klingman et al. (1974) developed a program NETGEN that solves capacitated and non-capacitated transportation problems, minimum cost flow network problems, and assignment problems. A basic solution to a classical capacitated transportation problem may contain more than m+n+P-2 positive values due to the capacity constraints which are additional to the m+n-1 independent equations in the simple case. Similarly, a basic solution to a solid capacitated transportation problem may contain constraints which are additional to the m+n+P-2 independent equation. Rao and Shaftel (1989b) designed an algorithm for solving a variety of non-capacitated and capacitated transportation production problems with a nonlinear objective function.

Consider m origins O_i (i=1, 2…..m) and n destinations D_j (j= 1, 2……n). At each origin O_i, let a_i be the quantity of homogeneous product which we want to transport to n destination D_j to satisfy the demands b_j there. The sources may be production facilities, warehouses, supply points etc., and the destinations may be consumption facilities, demand points, etc. A penalty c_{ij} is associated with transportation of a unit of the product from sources i to destination j for a two-dimensional transportation problem. A penalty c_{ij} is associated with transportation of a unit of the product from i-th sources to j-th destination by means of the p-th conveyance for a three-dimensional transportation problem. The penalty could represent transportation cost, delivery time, quantity of goods delivered, under used capacity, etc. A variable X_{ij} represents the unknown quality to be transported from origin O_i to destination D_j for a two-dimensional capacitated transportation problem. A variable X_{ij} represents the unknown quality to be transported from origin O_i to destination D_j by means of the pre- conveyances for a three-dimensional capacitated transportation problem. Let e_p be the capacity of the p-th conveyance. In real world, all the capacitated transportation problems are not single objective in nature. We may have more than one objective function in a transportation problem with capacity restrictions. Let r_{ij} be the capacity restrictions on route i,j for a two-dimensional capacitated transportation problem and r_{ijp} be the capacity restrictions on route I,j by means of the p-th conveyance for a three-dimensional capacited transportation problem.

Mathematical model of a multiobjective two-dimensional capacitated transportation problem can be stated as:

Minimize $Z_k = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^{k}\, x_{ij}, k = 1, \dots .K$ (1.1)

Subject to

$\sum_{j=1}^{n} x_{ij = a_1} \quad i = 1,2, \dots \dots m$ (1.2)

$\sum_{i=1}^{m} x_{ij} = b_j, \ j = 1,2, \dots \dots \dots n$ (1.3)

$0 \le x_{ij} \le r_{ij},\ i = 1,2, \dots \dots m; \quad j = 1, \dots \dots . n.$ (1.4)

Where subscript on Z_k and superscript on c_{ij}^k denote penalty criterion, $a_i > 0$ for all I, $b_j > 0$ for all j, $r_{ij} > 0$ for all i,j, and $\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$ is considered as the balanced condition.

A necessary condition for the existence of a feasible solution to the foregoing transportation problem is $\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$

However this condition is not sufficient because of condition (1.4)

Mathematical model of a multiobjective three-dimensional capacitated transportation problem can be stated as

Min $Z_k = \sum_{i=1}^m \sum_{j=1}^n \sum_{p=1}^P c_{ijp}^k x_{ijp}, k = 1, \dots \dots . . K$ (1.5)

Subject to

$$\sum_{j=1}^n \sum_{p=1}^P x_{ijp} = a_i, \quad i = 1,2, \dots \dots . m \quad (1.6)$$

$$\sum_{i=1}^m \sum_{p=1}^P x_{ijp} = b_j, \quad j = 1,2, \dots \dots . n \quad (1.7)$$

$$\sum_{i=1}^m \sum_{j=1}^n x_{ijp} = e_p, \quad p = 1,2, \dots \dots . P \quad (1.8)$$

$$0 \le x_{ijp} \le r_{ijp}\ for\ all\ i,j,p \quad (1.9)$$

Where subscript on Z_k and superscript c_{ijp}^k denote penalty criterion $a_i > 0\ for\ all\ i, b_j > 0\ for\ all\ j, e_p > 0\ for\ all\ p, r_{ijp} \ge 0\ for\ all\ i,j,p$ and $\sum_{i=1}^m a_i = \sum_{j=1}^n b_j = \sum_{p=1}^P e_p$ as balanced condition. This balanced condition is a necessary condition for the problem to have a feasible solution n; however this is not sufficient because of the condition (1.9)

The multiobjective solid capacitated transportation problem can be converted to a multiobjective classical capacitated transportation problem by taking p=1. This means, instead of considering different modes of transport, if a single conveyance is considered then the problem reduces to a classical capacitated transportation problem as:

Min $Z_k = \sum_{j=1}^n \sum_{p=1}^P x_{ijp} = a_1, k = 1,2, \dots \dots . K$ (1.10)

Subject to

$$\sum_{j=1}^n x_{ij1} = a_i, \quad i = 1,2, \dots \dots . m \quad (1.11)$$

$$\sum_{i=1}^m x_{ij1} = b_j, \quad i = 1,2, \dots \dots . m \quad (1.12)$$

$$\sum_{i=1}^m \sum_{j=1}^n x_{ij1} = e_1, \quad i = 1,2, \dots \dots . m \quad (1.13)$$

$$0 \le x_{ij1} \le r_{ij1}, i = 1,2, \dots . . m; j = 1 \dots \dots . ., n \quad (.14)$$

$$\sum_{i=1}^m a_i = \sum_{j=1}^n b_j = e_1 \quad (1.15)$$

Now setting $c_{ijp}^k = c_{ijp}^k$ for all i, j, k, $x_{ij1} = x_{ij}$ for all i, j and $r_{ij1} = r_{ij}$ for all i, j we show that this model is nothing but an exact formulation of the multiobjective classical capacitated transportation problem.

A multiobjective solid capacitated transportation problem arises due to the availability of heterogeneous conveyances for the shipment of goods.

There exit various ways of transforming a classical capacitated transportation problem into one of the ordinary type (Vajda (1961)) and Wagner (1959). Hassin and Zemel (1988) presented a classical capacitated transportation problem assuming the capacities as random variables. They proved assumption conditions on the supplies and demands which assure a feasible solution for the problem.

The solid transportation problem was first introduced by Haley (1963a) who gave a solution method of the single objective solid transportation problem. Patel and Tripathy (1989) discussed the variants of a single objective solid transportation problem. Misra and Das (1981a, 81b) considered the single objective solid transportation problem with capacity restrictions. Two-dimensional and three-dimensional multiobjective capacitated transportation problems can be treated as vector minimum problems. Fuzzy programming technique can be used to solve these problems. We have developed a Fortran program based on fuzzy programming technique to solve the multiobjective transportation problems. In the literature, it is observed that no work has been carried out on multiobjective classical and solid capacitated transportation problems.

II. FUZZY PROGRAMMING ALGORITHM FOR THE MULTIOBJECTIVE CAPACITATED TRANSPORTATION PROBLEM

We use the fuzzy programming technique to solve multiobjective two and three-dimensional capacitated transportation problems. Let L_k be the aspiration level of achievement and U_k be the highest acceptable level of achievement for the k-th objective, let $d_k = U_k - L_k$ be the degradation allowance for the k-th objective.

Step -1 solve the multiobjective two-dimensional capacitated transportation problem as a single objective two-dimensional capacitated transportation problem or the multiobjective three-dimensional capacitated transportation problem as a single objective three-dimensional capacitated transportation problem using each time, only one objective and ignoring all others. Repeat the process of K times.

Step-2 from the results of step-1 determine the corresponding values for every objective at each solution derived.

Step-3 From step-2 we find, for each objective, the best value (L_k) and the worst value (U_k) corresponding to the set of solutions. Initial fuzzy model is then established by the aspiration levels. The model can be represented as:

Find x_{ij}, i=1, ……., m; j=1,……n, so as to satisfy

$Z_k \leq L_k, k = 1,2, \dots \dots \dots . K$

And constraints (1.2), (1.3) and (1.4)

For both the two-dimensional multiobjective capacitated and the three-dimensional multiobjective capacitated transportation problems, the membership function $\mu_k(Z_k)$ corresponding to the k-th objective is defined as

$$\mu_k(Z_k) = \begin{cases} 1 & if\ Z_k \leq L_k \\ (\frac{Z_k - L_k}{U_k - L_k}) & if\ L_k < Z_1 < U_k \\ 0 & if\ Z_k \geq U_k \end{cases}$$

An equivalent single objective linear programming model of the two-dimensional multiobjective capacitated transportation problem can be represented as:

Subject to $\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^{k} x_{ijp} + \lambda(U_k - L_k) \leq U_k, k = 1, \dots \dots \dots . K$

And constraints (6.2), (6.3), (6.4), and $\lambda \geq 0$, where

$\lambda = \min_{k} \{\mu_k(Z_k)\}$

This LP problem can be solved by a linear programming algorithm to find an optimal compromise solution. By using the solution all the objective functions can be evaluated. Similarly, an equivalent linear programming model of the three-dimensional multiobjective capacitated transportation problem can be represented as:

Subject to $\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{P=1}^{P} c_{ij}^{k} x_{ijp} + \lambda(U_k - L_k) \leq U_k,$ k=1,2,………K and constraints (1.6), (1.7), (1.8), (1.9) and λ=0

This foregoing linear programming problem can be solved by a linear programming algorithm to find an optimal compromise solution. By using the solution, value of the objective functions can be calculated.

Example: To illustrate the fuzzy programming algorithm, we have considered a multiobjective two-dimensional capacitated transportation problem having three objective functions, three source point and three demand points all the necessary data are given below:

Supplies: $a_1 = 100, a_2 = 125, a_3 = 75$

Demands : $b_1 = 100, b_2 = 125, b_3 = 75$

Penalties:

C^1	D_1	D_2	D_3
O_1	3	1	−1
O_2	4	2	5
O_3	−1	6	4

C^2	D_1	D_2	D_3
O_1	2	4	3
O_2	5	6	4
O_3	3	−1	3

C^3	D_1	D_2	D_3
O_1	7	7	5
O_2	1	7	1
O_3	5	7	3

Capacity restrictions are given as:

$0 \le x_{11} \le 45,\quad 0 \le x_{12} \le 60,\quad 0 \le x_{13} \le 40,$
$0 \le x_{21} \le 90,\quad 0 \le x_{22} \le 110,\quad 0 \le x_{23} \le 80,$
$0 \le x_{31} \le 125,\quad 0 \le x_{32} \le 85,\quad 0 \le x_{33} \le 130,$

Applying fuzzy programming technique to the above problem, we obtain the upper bound and the lower bound of each objective as $U_1 = 1000, L_1 = 685, U_2 = 1150, L_2 = 930, U_3 = 1490, L_3 = 1280$ and compute optimal compromise solutions as:

$x_{11} = 16.691, x_{12} = 43.309, x_{13} = 40, x_{21} = 26.324$
$x_{22} = 18.676, x_{23} = 80, x_{31} = 16.985, x_{32} = 18.015$
$x_{33} = 40, \lambda = 0.485$

Optimal compromise values of the objectives Z_1, Z_2 and Z_3 are 847.1324, 1043.2353 and 1388.0882 respectively. Also we get three non-dominated solutions {685, 1150, 1360}, {1000, 930, 1490} and {865, 1054, 1280} for the three objectives.

III. CONCLUSIONS

For the multiobjective classical or solid capacitated transportation problem with K number of objective functions fuzzy programming technique gives K number of non-dominated solutions and an optimal compromise solution. Using fuzzy programming technique, one can easily find a compromise solution. A variety of methods, such as STEP method, utility function method lexicographic method, and interactive method have been developed by researchers for multiobjective linear programming problem. One may use these methods to solve a multiobjective capacitated transportation problem. However, all these methods are not straightforward for finding an optimal compromise solution. On the whole, fuzzy programming technique can be used to find an optimal compromise solution of a multiobjective classical or solid capacitated transportation problem.

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